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On the scattering of antiprotons in nuclear matter

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With 4 figures in the text

ABSTRACT

A statistical model calculation of the effective scattering cross section per nucleon in nuclei is presented. Because of the Pauli exclusion principle, the scattering cross section is reduced to $\sigma_{\text{eff.}} = f \cdot \sigma_{\text{sc.}}$; $\sigma_{\text{sc.}}$ being the free antiproton-nucleon cross section. The anisotropic form of the scattering cross section is taken into account. Formulae are given by which the reduction factor f may be evaluated. At 190 MeV kinetic energy (inside the nucleus) the computed value of the reduction factor is 0.62 for an assumed total cross section of 150 mb on free nucleons. The experimental situation is discussed.

Introduction

If we take the view that the inelastic scattering of high energy particles with a nucleus is caused by individual particle-nucleon collisions with a variety of momentum transfers, then the presence of other nucleons than the collision partner will act to reduce the cross section valid for the particle-free nucleon collisions. The reduction is caused by the fact that certain types of collisions are forbidden by the Pauli exclusion principle. The reduced cross section will in this paper be called the effective scattering cross section per nucleon or simply σ_{eff}. The results can also be expressed in terms of a reduction factor, f, defined by the relation $\sigma_{\text{eff.}} = f \cdot \sigma_{\text{sc.}}$.

The basic paper on this subject was written by M. L. Goldberger [1]. He treated the general problem and solved the equations for the case of high energy neutron scattering with the assumption of an isotropic cross section in the center of mass frame of reference (C.M.S). His formula is

$$f = \left(1 - \frac{7}{5\,P_0^2}\right)$$

valid for projectile momenta $P_0 > \sqrt{2}$ in units of the maximum Fermi momentum. Goldberger also performed a Monte Carlo calculation for the more realistic case of an anisotropic cross section.

The corresponding problem for K-meson scattering was treated by Bhowmik *et al.* [2], also for the case of an isotropic scattering cross section in C.M.S. Their result for projectile momenta $P_0 \geqslant 1$ in units of the maximum Fermi momentum is

$$f=\left(1-\frac{2}{5P_0^2}\right).$$

The present paper is concerned with the scattering of antiprotons in nuclear matter. It is known from experiments that the cross section for antiproton-free proton scattering is far from isotropic. It is in fact strongly peaked forward. The preference for forward scattering angles means a preference for small momentum transfers. This circumstance must lead to a stronger reduction of the cross section than in the case of isotropy. Therefore, the cross section on free nucleons must not be assumed to be isotropic. This fact causes the calculations to get rather involved and tedious.

In the first part of the paper, a recapitulation of Goldbergers general treatment is given. The notation used in this paper is introduced in this section. Then follows a section in which the antiproton-free nucleon scattering cross section is reviewed. The main part of the paper consists of a calculation of the effective scattering cross section and the corresponding reduction factor for antiprotons in nuclear matter. It should be mentioned that the calculations have been carried out in the nonrelativistic approximation.

General formulation of the problem

The protons (and the neutrons) in the nucleus are described in momentum space by a sphere filled to the maximum Fermi momentum, P_F. The density within the sphere is constant (n) and is zero outside the sphere. The maximum Fermi momentum will be taken as 210 MeV/c. In the following all momenta will be given in units of P_F.

Notation used:

P_0 = momentum of incoming antiproton (inside the nucleus)
g = transferred momentum during a collision
η = angle between the vectors P_0 and $-g$
p_0 = center of mass momentum before the collision
p_f = center of mass momentum after the collision
θ = scattering angle in center of mass system
P_1 = momentum of target nucleon.

Following Goldberger we take the z-axis of the cordinate system along g and get the situation as in Fig. 1.

All momenta are resolved in cylindrical coordinates:

$$\begin{aligned}&P_1:(\varrho, z, \phi)\\&P_0:(P_0\sin\eta, -Z, 0) \qquad Z=+P_0\cos\eta\\&g\ \ :(0, g, 0)\end{aligned}$$

The differential cross section in the C.M.S. is generally written as a function of energy and angle, and is $\sigma(p_0,\theta)\,d\Omega$. The differential cross section on protons is σ_p, on neutrons σ_n. Denoting the effective cross section per nucleon by σ_{eff} (A = massnumber), one gets according to Goldberger

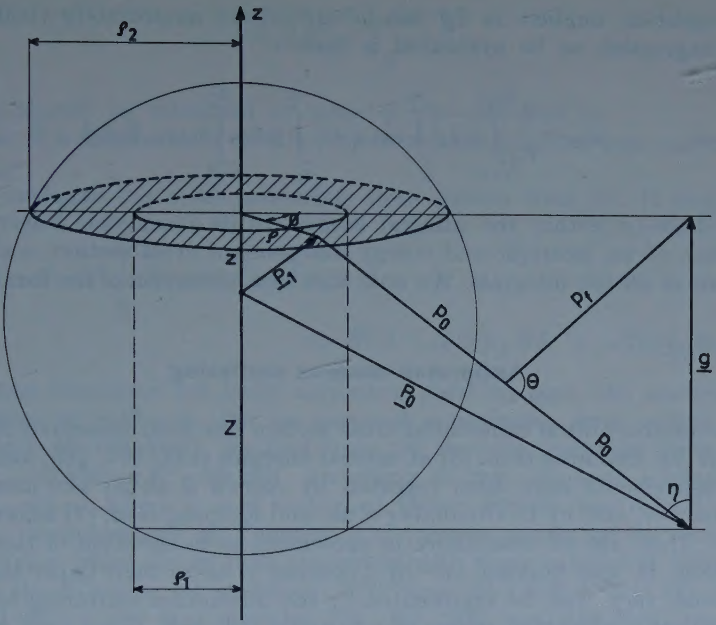

Fig. 1. The sphere with unit radius represents the filled Fermi sphere in momentum space. Collision situations among the target nucleons allowed by the Pauli principle are found in the shaded area in the plane $z=g-Z$.

$$\sigma_{\text{eff}} A=\frac{1}{P_0}\int d\boldsymbol{g}\int d\boldsymbol{P}_1 n_p 2 p_0 \sigma_p(p_0,\theta)\frac{1}{p_0^2}\delta(p_0-p_f)+$$
$$+\text{(corresponding expression for neutrons)} \qquad (1)$$

where the δ-function takes care of energy conservation. As there are reasons to assume $\sigma_p=\sigma_n$, we will in the following make this assumption. In eqn. (1) $d\boldsymbol{P}_1=\varrho\, d\varrho\, dz\, d\phi$ and $d\boldsymbol{g}=g^2 dg \sin\eta\, d\eta\, d\psi$, where ψ is the azimuth angle. Because of energy and momentum conservations and for $\boldsymbol{g}$ and η fixed, contributions come only from target nucleons which have the end points of their P_1-vectors lying in a plane perpendicular to $\boldsymbol{g}$ as is shown in Fig. 1. After a collision allowed by the Pauli exclusion principle, the endpoint of the new target nucleon vector must lie outside the Fermi sphere. This leads to contributions only from the shaded area in Fig. 1. The limits of integration for ϱ are therefore given by $\varrho_1=\sqrt{1-Z^2}$, and $\varrho_2=\sqrt{1-(g-Z)^2}$ when $Z\leqslant 1$, and given by $\varrho_1=0$ and $\varrho_1=\sqrt{1-(g-Z)^2}$ when $Z\geqslant 1$. The variables Z and η are connected by $Z=P_0\cos\eta$, which shows that the upper limit on Z is given by $Z=P_0$.

After carrying out the z-integration as in Goldbergers treatment (which leads to the abovementioned restriction to the plane $z=g-Z$), we get

$$\sigma_{\text{eff}}=\frac{3}{\pi P_0}\int g^{-1}d\boldsymbol{g}\int_{\varrho_1}^{\varrho_2}\varrho\, d\varrho\int_{\phi=0}^{2\pi}\sigma(p_0,\theta)\, d\phi \qquad (2)$$

The ψ-integration implicit in dg can be carried out immediately yielding a factor 2π. The expression to be evaluated is then

$$\sigma_{\text{eff}} = \frac{6}{P_0} \int\limits_{g=0}^{P_0+1} g\,d\,g \int\limits_{\eta_1}^{\eta_2} \sin\eta\,d\,\eta \int\limits_{\varrho_1}^{\varrho_2} \varrho\,d\,\varrho \int\limits_{\phi=0}^{2\pi} \sigma(p_0, \theta)\,d\,\phi \tag{3}$$

We must leave σ within the integral because of its anisotropic behavior. For the simpler case of an isotropic and energy independant cross section, σ can be taken out in front of all the integrals. We now turn to a discussion of the form of $\sigma\,(p_0, \theta)$.

Antiproton-nucleon scattering

The antiproton-proton differential cross section has been measured in a counter-experiment by Coombes *et al.* [5] at several energies (133, 197, 265, and 333 MeV). Other measurements have been reported by Agnew *et al.* [6] who used a propane bubble chamber, and by G. Goldhaber *et al.*, and Ekspong *et al.* [7] who used nuclear emulsions. They are all essentially in agreement as to the form of the differential cross section. It was pointed out by Coombes *et al.* in their paper that the cross section could very well be represented by the diffraction scattering formula valid for a completely absorbing sphere of a suitable radius R. Using this, we can write $\sigma(p_0, \theta)$ in the following way in the present notation:

$$\sigma(p_0\,\theta) = p_0^2 R^2 \left[\frac{J_1\,(\hbar^{-1} R\,2\,p_0 \sin\frac{\theta}{2})}{2\,p_0 \sin\theta/2}\right]^2 \tag{4}$$

where J_1 is the Bessel function. This formula does probably not represent the large angle and especially the backward scattering correctly, because at high momentum transfers the details of the interaction enters. However, all that is known experimentally about the backward scattering is that it is small compared to the forward scattering [6, 7]. This holds also for the cross section given by eqn. (4), although it probably gives a too small backward scattering. An estimate of the systematic error hereby introduced in the reduction factor f shows that it may amount to about $-5\,\%$. It seems reasonable, therefore, to accept (4) as a good approximation to the actual cross section.

No experimental data exist on the differential neutron scattering cross section. According to Chamberlain *et al.* [7] who measured the total and annihilation cross sections on deutrons and protons at 457 MeV, it seems to be justified to put the total scattering cross sections on protons and neutrons equal. Guided by general principles about diffraction and the calculations published by Fulco [3] based on the Chew and Ball [4] theory, we will further assume that the differential cross sections are the same.

Turning now to our formula (4), we note that $2\,p_0 \sin\dfrac{\theta}{2} = g$, which is the transferred momentum. The nice feature about expression (4) is that the θ-dependance can be extracted out as a g-dependance. This circumstance makes the integration of (3) more attractive. We thus have

$$\sigma = p_0^2 R^2 \frac{J_1^2(g R \hbar^{-1})}{g^2} \tag{5}$$

where $R^{-1}\hbar$ should be measured in units of $P_F = 210$ MeV/c.

The radius R is chosen such that the measured annihilation cross section is given by $\pi R^2 = \sigma_a$.

We now evaluate the total scattering cross section from (5). It is given by

$$\sigma_{sc} = \int \sigma \, d\Omega = 2\pi R^2 \int_0^{2p_0} J_1^2(g R \hbar^{-1}) g^{-1} dg =$$

$$= \pi R^2 [1 - J_0^2(2 p_0 R \hbar^{-1}) - J_1^2(2 p_0 R \hbar^{-1})] \tag{6}$$

As the Bessel functions for large arguments tend to zero, the scattering cross section is close to $\pi R^2 = \sigma_a$. For an assumed annihilation cross section of 80 mb and an energy of 190 MeV, we find from (6) that the scattering cross section is given by $\sigma_{sc} = 0.866 \times \pi R^2 = 69$ mb. According to this, we have a relation of the form $\sigma_a + \sigma_{sc} = b\,\sigma_a$, where b is a numerical factor slightly less than 2. Strangely enough, the measurements by Coombes *et al.* [5] give values for the corresponding numerical factor which are in close agreement with those calculated from (6). This is shown by the following set of values: 1.86 (from (6) 1.84), 1.83 (1.87), 1.76 (1.88) and 1.85 (1.90) at the four energies 133, 197, 265, and 333 MeV, respectively. There is, however, probably no other significance in this close agreement than that most of the scattering is diffraction scattering.

Evaluation of the effective cross section

From Fig. 1. we find that p_0 is given in terms of the coordinates introduced by

$$(2 p_0)^2 = \varrho^2 - 2\varrho \cos\phi \, P_0 \sin\eta + P_0^2 \sin^2\eta + g^2 \tag{7}$$

This expression together with our assumed cross section in (5) is now introduced into (3), yielding an expression where the Bessel function part of the cross section can be moved out to the g-integral:

$$\sigma_{\text{eff.}} = \frac{6 R^2}{P_0} \int J_1^2(g R \hbar^{-1}) g^{-1} dg \int \sin\eta \, d\eta \int \varrho \, d\varrho \int_0^{2\pi} p_0^2 \, d\phi \tag{8}$$

By performing the ϕ-integration first, we achieve that the term containing $\cos\phi$ in (7) drops out, leaving the expression

$$\sigma_{\text{eff.}} = 3\pi R^2 P_0^{-1} \int_{g_1}^{g_2} J_1^2 g^{-1} dg \int_{\eta_1}^{\eta_2} \sin\eta \, d\eta \int_{\varrho_1}^{\varrho_2} \varrho (\varrho^2 + P_0^2 \sin^2\eta + g^2) \, d\varrho \tag{9}$$

The limits ϱ_1 and ϱ_2 were given earlier. The ϱ-integration is trivial. For the g and η integrations the limits are as given in Fig. 2, where η has been substituted by the relation $Z = P_0 \cos\eta$. We note that four regions of integration are given in

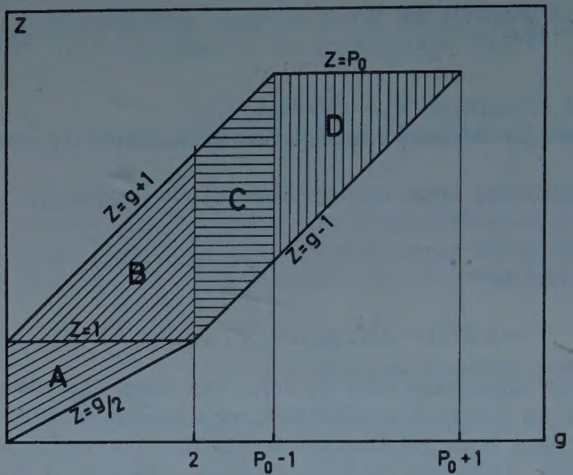

Fig. 2. Diagram showing the four regions of integration (A, B, C, and D) in the g, Z-plane. The limits of integration in Z and g are given.

Fig. 2, namely the regions A, B, C, and D. For the case $P_0 > 1$, which corresponds to antiproton energies above 24 MeV, the results are given below. In case $P_0 < 3$, region C contributes negatively. This means that the same formula may be used even for this case. The Pauli exclusion principle does not restrict the final momentum of the antiproton, contrary to the case of neutron scattering. We have carried out all the integrations, which are elementary, with exception for the integration over g, and obtained:

$$\sigma_{\text{eff.}} = \pi R^2 \frac{3}{4P_0^2} \left\{ \int_{g=0}^{2} dg\, J_1^2 f_1(g) + \int_{2}^{P_0-1} dg\, J_1^2 f_2(g) + \int_{P_0-1}^{P_0+1} dg\, J_1^2 f_3(g) \right\} \tag{10}$$

where the functions f_1, f_2, and f_3 are as follows:

$$f_1(g) = 2P_0^2 - \frac{4}{3}g - \frac{P_0^2 - 7}{6}g^2 - \frac{13}{120}g^4 \tag{11}$$

$$f_2(g) = \frac{8}{15g}(1 + 5P_0^2) \tag{12}$$

$$f_3(g) = \frac{2}{5}g^4 - P_0 g^3 + \frac{2}{3}(P_0^2 - 1)g^2 - \frac{1}{15g}(P_0 - 4)(P_0 + 1)^4 =$$

$$= \frac{1}{15g}[g - (P_0 + 1)]^2 [6g^3 - 3(P_0 - 4)g^2 - 2(P_0 - 4)(P_0 + 1)g - (P_0 - 4)(P_0 + 1)^2] \tag{13}$$

The effective cross section in (10) is a function of energy (or momentum P_0) with the free nucleon cross section as a parameter (through R). It should be emphasized that P_0 is the momentum of the antiproton *inside* the nucleus. It therefore depends on the energy outside and the potential.

For comparison, the effective cross section is also given for the hypothetical case of an isotropic scattering cross section, namely

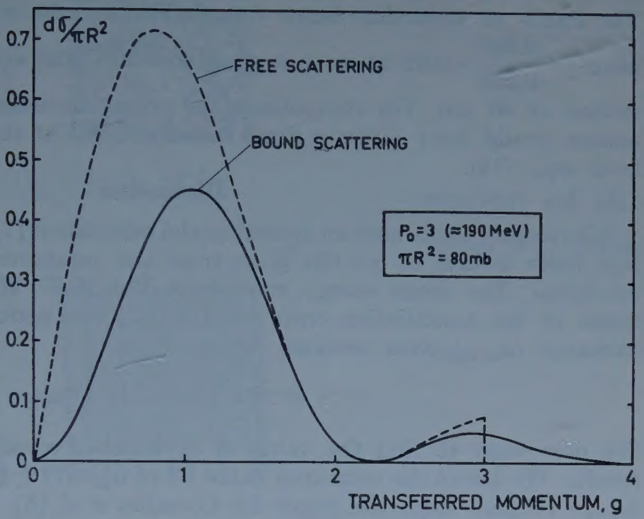

Fig. 3. Solid curve is for scattering on bound nucleons. Dashed curve is for scattering on free protons. The curves are computed for an antiproton momentum = 3 × the maximum Fermi momentum (i. e. 190 MeV). The transferred momentum, g, is in units of the maximum Fermi momentum. (The ordinate should read $\dfrac{1}{\pi R^2}\dfrac{d\sigma}{dg}$)

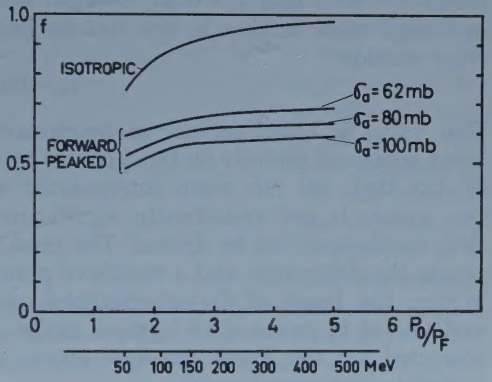

Fig. 4. The ratio $f=\sigma_{\text{eff}}/\sigma_{\text{sc}}$ as a function of antiproton momentum, P_0, in units of the maximum Fermi momentum, P_F. Inserted is an energy scale, based on $P_F=210$ MeV/c. The three curves, labelled "forward peaked", are computed from eqn. (10) and (6) for three absorption cross sections (62, 80, and 100 mb). The upper curve, labelled "isotropic," would be valid if the scattering cross section was isotropic in the center of mass system, eqn. (14).

$$\sigma_{\text{eff.}}=\sigma_{\text{sc}}\left(1-\frac{3}{5P_0^2}\right) \tag{14}$$

The final integration of (10), which involves the Bessel function, has been carried out numerically. The solid curve in Fig. 3 shows as an example the integrands of (10) as a function of the transferred momentum g. The graph is computed for $P_0=3$, which corresponds to an energy of 190 MeV, and for $\pi R^2=80$ mb. The dashed curve in the same figure shows the corresponding contributions to the cross section for the case of free proton scattering. From these two graphs it is clear that the cross section is strongly reduced in the region of small momentum transfers. The reduction becomes negligible for momentum transfers larger than 1.5 times the maximum Fermi momentum. The upper limit for g is $g=P_0$ for the free nucleon case, and is $g=P_0+1$ for the bound case. The evaluation of the integrals yield:

Bound nucleon scattering $\sigma_{\text{eff}}=0.540\times\pi R^2$
Free nucleon scattering $\sigma_{\text{sc}}=0.866\times\pi R^2$

The Pauli- or reduction factor f is defined by the relation $\sigma_{\text{eff}} = f \cdot \sigma_{\text{sc}}$. We thus have $f = \frac{0.540}{0.866} = 0.62$ at an energy of 190 MeV and an assumed annihilation cross section of 80 mb. For comparison, the erroneous assumption of an isotropic cross section would have given a Pauli factor of 0.935 at the same energy. This follows from eqn. (14).

Discussion

We recently performed an optical model calculation [7] which gave a total attenuation cross section of $\sigma = 150^{+27}_{-22}$ mb from the measured mean free path in nuclear emulsions. The mean energy was about 170 MeV. We write this cross section in terms of the annihilation cross section (σ_a), the scattering (σ_{sc}), and the charge exchange ($\sigma_{\text{ch ex}}$) cross sections, as

$$\sigma = \sigma_a + f_1 \sigma_{\text{sc}} + f_2 \frac{Z}{A} \sigma_{\text{ch ex}} \tag{15}$$

We now want to find the value of σ_a which is consistent with the experimental results. We insert the measured value [7] of $\sigma_{\text{sc}} = 77 \pm 17$ mb and take the value of $\sigma_{\text{ch ex}} = 10^{+2}_{-3}$ mb from the paper by Coombes *et al.* [5]. Then we use our computed factor $f_1 = 0.62$ and $f_2 = 0.90$, because according to Fulcos calculation the charge exchange cross section is not too far from isotropic. We then get an annihilation cross section

$$\sigma_a = 98^{+30}_{-25}\,\text{mb}$$

This value is based on our measurements on complex nuclei. The corresponding value measured directly on free protons by Coombes *et al.* [5] is 84^{+14}_{-12} mb at an energy of 133 MeV (80 mb when interpolated at 170 MeV). The difference between the two values is not statistically significant. One has to await better values before firm conclusions can be drawn. The presence of an attractive potential would increase the difference, and a repulsive potential would make the agreement better. Within the limits of the experimental errors, the results on complex nuclei can be understood in terms of an optical model when the free nucleon cross sections are corrected for the Pauli principle effect, and when the neutron and proton cross sections are assumed equal.

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